E 3981 (Pages : 3) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2016

Third Semester

Complementary Course—Statistics—PROBABILITY DISTRIBUTION

(Common to B.Sc. Mathematics, Physics and Computer Applications (3 Main)

[2013 Admission onwards]

Time : Three Hours Maximum : 80 Marks

Part A (Short Answer Question)

Answer **all** *questions.*
1 *mark each.*

1. Define a binomial distribution.
2. Define memorylessness property.
3. Define an exponential distribution.
4. State weak law of large numbers.
5. Define Standard error.
6. For a Poisson distribution mean = 3, find $P(X=0)$.
7. Find the mode of the geometric distribution $p(x)=\left(\frac{1}{2}\right)^x$, $x=1,2,3,.....$
8. If $X \sim N(0,1)$, then find $p(1\times 1\le 1)$.
9. Find the variance of a uniform r.v. in the internal (3, 4).
10. What do you mean by sampling distribution ?

$(10 \times 1 = 10)$

Part B (Brief Answer Questions)

Answer any **eight** *questions.*
2 *marks each.*

11. Obtain the mean of binomial distribution with parameters (n, p).
12. Obtain the m.g.f. of continuous rectangular distribution in the internal (0, 2).

Turn over

13. If $X \sim N(25, 9)$ and $Y \sim N(20, 4)$ and if X and Y are independent, find the distribution of $X + Y$.

14. Derive the additive property of gamma distribution.

15. Define convergence in probability and Bernoulli's law of large numbers.

16. If $X \sim N(16, 4)$, find an upper bound for $P(|X - 16| > 6)$ using Chebychev's inequality.

17. Show that for a Poisson distribution with unit mean, M.D. about mean is $\frac{2}{3}$.

18. Find the mean and variance of an exponential distribution with parameter θ.

19. Show that a linear combination of independent normal variation is also a normal variate.

20. Define beta distribution of the first kind and obtain the harmonic mean.

21. Examine whether WLLN holds for the sequence $\{X_n\}$ if independent r.vs. where

$$P\left\{X_n = \frac{1}{\sqrt{n}}\right\} = \frac{2}{3}.$$

$$P\left\{X_n = \frac{-1}{\sqrt{n}}\right\} = \frac{1}{3}.$$

22. Define a Chi-square statistic. What is the distribution of sample variance of a sample of size 'n' drawn from $N(\mu, \sigma^2)$.

$(8 \times 2 = 16)$

Part C (Description/Short Essay)

Answer any **six** *questions.*
4 marks each.

23. Derive the recurrence relation connecting central moments of a binomial distribution.

24. Obtain the skewness and Kurtosis coefficients for a Poisson distribution.

25. If a r.v.X has density function $f(x) = e^{-x}$, $0 < x < \infty$, show that Chebychev's inequality gives $P\{|X - 1| > 2\} < \frac{1}{4}$ and show that the actual probability is e^{-3}.

26. Obtain the sampling distribution of mean drawn from $N(\mu, \sigma^2)$.

27. Obtain the relationships between χ^2, t and F distributions.

28. In a large group of students, 5 % are under 150 cms. and 40 % are between 150 cm. and 162 cm. in height. Assuming normal distribution, find mean and S.D. of height.

29. Show that 't' distribution tends to normal distribution under certain conditions.

30. Obtain the m.g.f. of a χ^2 variate and hence obtain its mean and variance.

31. Use CLT to find the least value of n if we require $P(\bar{X} > \mu + 0.281\,\sigma) = 0.05$.

(6 × 4 = 24)

Part D (Essay)

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) Define normal distribution. What are its important properties ?

    (b) Obtain the m.g.f. of X ~ N (0, 1).

33. (a) Define 't' random variable and obtain the expression for its even ordered central moments.

    (b) Show that $\frac{\bar{X} - \mu}{s / \sqrt{n-1}} \sim t_{(n-1)}$

    While sampling from $N(\mu, \sigma^2)$. s^2 is the sample variance.

34. State and prove Chebychev's inequality. What are its advantages ?

35. State and prove Lindeberg–Levy's form of CLT.

(2 × 15 = 30)